

Optimal Workforce Configuration Incorporating Absenteeism and Daily Workload Variability

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Abstract—This paper is one in a series that introduces concepts of Just-in-Time-Personnel. Management of worker job time and assignment is in many ways analogous to inventory management. Idle workers represent unutilized, “inventoried” personnel, imposing potentially large costs on management. But a lack of workers when needed may force the use of otherwise unnecessary overtime or other emergency procedures, creating excessive costs analogous to costs of stockout in traditional inventory systems. A system having Just-in-Time-Personnel attempts to meet all demands for personnel at minimum cost by sharply reducing both excess worker inventory with its concomitant “paid lost time” and underage of worker inventory with its associated costs of stockout. In this paper, a workforce is assumed to comprise three types of equally proficient worker: full timers, scheduled part times and call up temporaries. Each day the facility employing the workers must perform an amount of work that varies randomly from day to day, according to a normal distribution. If, due to random absenteeism, insufficient numbers of scheduled workers appear on a given day, management must employ overtime and/or temporaries to complete the workforce complement. For large facilities, we formulate this problem mathematically and develop an algorithm for management to optimally configure the workforce, where the design criterion is minimization of average daily cost, and where certain reasonable management-stated constraints limit the number of design options available.

BACKGROUND AND MOTIVATION

In configuring a workforce, management is usually faced with deciding the appropriate mix of employees. Often, the options include both full time and scheduled part time employees, as well as temporary part time employees who may be on call. The decision problem is made complex by the random nature of employee absences and by day-to-day fluctuations in workload. Costs accrue due to the fixed payroll associated with scheduled employees, the variable costs of part-time temporaries and the variable costs of overtime.

In this paper, we model the workforce configuration problem in an aggregate and simple way that should be suitable for applications in large manufacturing and service firms. The objective is to optimally size each of the components of the workforce, where the optimality criterion is average cost (both fixed and variable). The problem was motivated by a workforce planning study undertaken by the authors with a major U.S. logistics services firm having more than 100,000 employees.

ASSUMPTIONS

We assume that there are three groups of employees in the workforce: (1) N_1 full time employees (FTEs); (2) N_2 scheduled part time employees (PTEs); and (3) a potentially infinite pool of call up temporary employees (TEMPs). Any employee who shows up on a given day works a full 8 hour shift.

Absenteeism is a regular and expected occurrence in day-to-day operations. In this model, we assume that any particular scheduled employee will not appear for work on any given day with probability p , independent of appearances on previous days and independent of the absenteeisms of fellow employees.

Fluctuation in daily workload is also a standard feature of operations. To compensate for statistical variations in daily work levels and in employee attendance, management can use either overtime or call up temps. We assume that (1) each day's work must be done on that day (i.e. there is no backlogging of work for subsequent days), and that (2) each day's workload becomes known to management no later than the beginning of that workday, thereby allowing time to call up temps, if necessary.

We model daily workload D at the facility, measured in *hours of work to be performed*, as a normal or Gaussian random variable with known mean μ_D and known variance σ_D^2 . The work can be performed by any combination of employee types, with equal proficiency by each type. (In applications, μ_D and σ_D^2 may vary by day of week and season of year. While we do not explicitly include this time dependency, our model could easily be extended to include such realistic complications. Too, the role of temporary and call-up personnel can be more pronounced during days and seasons projected to have higher than average workload.)

In the current paper, we present two formulations of our model. In the first, we find the optimal mix of workers so as to minimize the cost of workers subject to constraints on the ratio of full and part time workers and an upper bound on the total hours of overtime. In the second formulation, we introduce a management requirement limiting the fraction of days that overtime and/or TEMPs may be used. Finally, we denote by C_f , C_p , C_t and C_{OT} the hourly wage rates of FTEs, PTEs, TEMPs and overtime, respectively.

The model herein is but one representation of the use of "pooled" temporary workers in the work force. For a comprehensive taxonomy, discussion of additional models, and a comprehensive set of additional references we recommend the Ph.D. thesis by Leegwater [1]. Also, for application of similar ideas in the urban transportation industry see Wilson [3].

THE MODEL

On any given day some of the scheduled workers may be absent due to illness, personal considerations, or other reasons. Let the random variable S be the total number of scheduled worker hours, both full time and part time, that are available on a given day. That is, from the potential base level of $(N_1 + N_2) * 8$ scheduled worker hours, S represents the residual worker hours available on a given day after subtracting hours of scheduled but absent workers. Let $Y_i = 1$ if scheduled worker i appears for work on any given day while $Y_i = 0$ if worker i is absent, $i = 1, \dots, N_1 + N_2$. The Y_i correspond to independent Bernoulli trials with parameter p , which is the probability that a randomly selected worker is absent from work on any given day. It follows that the total number of available hours of work by scheduled workers on any given day is $8 \sum_{i=1}^{N_1 + N_2} Y_i = S$. Since $S/8$ is a binomial random variable with parameters $1 - p$ and $N_1 + N_2$, S has mean $\mu_S = 8(N_1 + N_2)(1 - p)$ and variance $\sigma_S^2 = 64(N_1 + N_2)(1 - p)p$. Since we are focusing on large facilities, we can assume that the cumulative distribution of S can, by the Central Limit Theorem, be closely approximated by the Gaussian or normal distribution with the aforementioned mean and variance.

Since both S and the daily workload D are normal random variables, so is their difference, $D - S$, which, when positive, represents the total number of overtime and/or temp hours required to meet today's workload. The mean μ and variance σ^2 of $D - S$ are given by:

$$\mu = \mu_D - 8(N_1 + N_2)(1 - p) \quad (1)$$

$$\sigma^2 = \sigma_D^2 + 64(N_1 + N_2)(1 - p)p. \quad (2)$$

THE PROBLEMS

The total number of overtime and temp hours required on any given day is denoted by the random variable NT , given by:

$$NT = \max(D - S, 0).$$

The mean of NT is:

$$\begin{aligned}\overline{NT} &= E[\max(D - S, 0)] = \int_0^\infty (d - s) dF_{D-S} \\ &= \sigma L\left(\frac{-\mu}{\sigma}\right)\end{aligned}\quad (3)$$

where $L(z)$ is the standardized loss function defined as:

$$L(z) = \int_z^\infty (x - z)\phi(x) dx$$

and where $\phi(x)$ is the probability density function of a standard normal random variable. (Tables of $L(z)$ are available in many books, e.g. [2].)

Let us denote \bar{N}_3 as the expected number of temps and \overline{OT} as the expected number of overtime hours. Obviously, $\overline{NT} = \overline{OT} + 8\bar{N}_3$.

Suppose that there are two major constraints for the problem. The first is to ensure that the number of full time workers is at least a fraction, γ , ($0 < \gamma \leq 1$) of the total number of scheduled workers, both full time and part time. This type of condition is often found in negotiated labor/management contractual agreements. (The contract between union-represented U.S. postal workers and the management of the U.S. Postal Service is an example.) The second constraint stipulates that the expected amount of overtime utilized is at most a fraction, δ , ($0 \leq \delta \leq 1$) of the total number of hours of scheduled full timers ($8N_1$). This constraint, too, may be part of a labor/management agreement, or it may simply be a management objective.

Now we can consider the problem of finding values for N_1 , N_2 , \bar{N}_3 and \overline{OT} that minimize the expected total daily cost of the system. We call the problem P_1 :

$$\begin{aligned}\min \quad & C = 8N_1C_f + 8N_2C_p + 8\bar{N}_3C_t + \overline{OT}C_{OT} \\ \text{s.t.} \quad & N_1 \geq \gamma(N_1 + N_2) \\ & \overline{OT} \leq \delta 8N_1 \\ & 8\bar{N}_3 + \overline{OT} = E[\max(D - S, 0)] \\ & N_1, N_2, \overline{OT}, \bar{N}_3 \geq 0.\end{aligned}\quad (P_1)$$

The second problem is similar to the first but with one additional constraint, requiring that no more than a fraction of $1 - \alpha$ work days can employ overtime and/or temps to complete the day's work, i.e.

$$P\{D - S < 0\} = \alpha. \quad (4)$$

As above, this constraint might also be part of a larger labor/management agreement, with labor possibly desiring an intermediate "modestly positive" value of α . This would ensure some overtime compensation for its members but also guarantee that total union membership will not be lower than desired because of excessive use of overtime. Of course, depending on the relative hourly cost figures, management may strongly prefer the use of temps to use of overtime.

Since $(D - S - \mu)/\sigma$ is a standard normal random variable, eqn (4) yields:

$$\frac{-[\mu_D - 8X(1 - p)]}{\sqrt{\sigma_D^2 + 64X(1 - p)p}} = Z_\alpha, \quad (5)$$

where $X = (N_1 + N_2)$ and Z_α is the value such that for a standard normal random variable Z , $P(Z \leq Z_\alpha) = \alpha$. Equation (5) yields a quadratic equation in X having two solutions:

$$X = \frac{\mu_D + 4Z_\alpha^2 p \pm Z_\alpha \sqrt{8p\mu_D + 16Z_\alpha^2 p^2 + \sigma_D^2}}{8(1 - p)}. \quad (6)$$

If $\alpha < 0.5$, then $Z_\alpha < 0$ and therefore from (5), $X < \mu_D/8(1-p)$. We thus choose the solution:

$$X = \frac{\mu_D + 4Z_\alpha^2 p + Z_\alpha \sqrt{8p\mu_D + 16Z_\alpha^2 p^2 + \sigma_D^2}}{8(1-p)}. \quad (7)$$

If $\alpha > 0.5$, then $Z_\alpha > 0$ and therefore again from (5), $X > \mu_D/8(1-p)$; we thus choose the same solution as in (7). If $\alpha = 0.5$, we choose $X = \mu_D/8(1-p)$.

Now we can present the second problem, P_2 :

$$\begin{aligned} \min \quad & C = 8N_1 C_f + 8N_2 C_p + 8\bar{N}_3 C_t + \overline{OT} C_{OT} \\ \text{s.t.} \quad & N_1 \geq \gamma(N_1 + N_2) \\ & \overline{OT} \leq \delta 8N_1 \\ & 8\bar{N}_3 + \overline{OT} = E[\max(D - S, 0)] \\ & P\{D - S < 0\} = \alpha \\ & N_1, N_2, \bar{N}_3, \overline{OT} \geq 0. \end{aligned} \quad (P_2)$$

SOLVING PROBLEMS P_1 AND P_2

We first consider problem P_1 . Let X_{\max} be the smallest total number of full and part time workers needed if we require that $P\{D - S < 0\} \approx 1$ (almost equal to 1). From eqn (7), with $Z_\alpha = Z_1 = 4$, we obtain:

$$X_{\max} = \frac{\mu_D + 64p + 4\sqrt{8p\mu_D + (16)^2 p^2 + \sigma_D^2}}{8(1-p)}. \quad (8)$$

Given these developments, we can now present the following simple procedure for solving P_1 :

Step 0: Let $X = N_1 + N_2$. Set $X = 0$ and $C^* = \infty$.

Step 1: a. If $C_f \geq C_p$, set $N_1 = \gamma X$ and $N_2 = (1 - \gamma)X$.

If $C_f < C_p$, set $N_1 = X$ and $N_2 = 0$.

b. Calculate μ and σ according to formulas (1) and (2).

c. If $C_{OT} \leq C_t$, set

$$\overline{OT} = \int_0^{\delta 8N_1} (d - s) dF_{D-S} = \sigma L\left(\frac{-\mu}{\sigma}\right) - \sigma L\left(\frac{\delta 8N_1 - \mu}{\sigma}\right) - \delta 8N_1 \left(1 - F\left(\frac{\delta 8N_1 - \mu}{\sigma}\right)\right) \quad (9)$$

and

$$\bar{N}_3 = \frac{\sigma L\left(\frac{-\mu}{\sigma}\right) - \overline{OT}}{8}. \quad (10)$$

If $C_{OT} > C_t$, set

$$\bar{N}_3 = E[\max(D - S, 0)] = \frac{\sigma L\left(\frac{-\mu}{\sigma}\right)}{8} \quad \text{and} \quad \overline{OT} = 0.$$

d. Calculate the objective function value C . If $C < C^*$, set $C^* = C$ and set

$$N_1^* = N_1, N_2^* = N_2, \bar{N}_3^* = \bar{N}_3 \quad \text{and} \quad \overline{OT}^* = \overline{OT}.$$

Step 2: If $X < \lfloor X_{\max} \rfloor$ set $X = X + 1$ and go to Step 1. Otherwise,

$N_1^*, N_2^*, \bar{N}_3^*$, and \overline{OT}^* are optimal with objective function value C^* .

We note that, based on all our calculations, C is a convex function. There is thus no need to calculate X for all values between 0 and X_{\max} . The first time that we find C increasing, we can stop.

Now consider problem P_2 . The procedure for solving P_2 is similar to that for P_1 :

Step 0: Calculate \hat{X} according to (7), viz.

$$\hat{X} = \frac{\mu_D + 4Z_\alpha^2 p + Z_\alpha \sqrt{8p\mu_D + 16Z_\alpha^2 p^2 + \sigma_D^2}}{8(1-p)}.$$

Step 1: Identical to Step 1 for P_1 except for (d) which we need not check if $C < C^*$.

Finally, we note that, given $X^* = N_1^* + N_2^*$ of P_1 , we can calculate that corresponding α giving us the same solution in P_2 . We refer to this α value as the “implied α ”. Using (5), the implied α , $\hat{\alpha}$, is such that

$$Z_{\hat{\alpha}} = \frac{[\mu_D - 8X^*(1-p)]}{\sqrt{\sigma_D^2 + 64X^*(1-p)p}}.$$

NUMERICAL EXAMPLE

As an example, consider a service facility for which $C_t = \$20/\text{h}$, $C_p = \$18/\text{h}$, $C_o = \$30/\text{h}$ and $C_{OT} = \$27/\text{h}$. We also assume that $\mu_D = 400$, $\sigma_D^2 = 900$, $p = 0.15$, $\gamma = 0.8$ and $\delta = 0.2$. Since $C_p < C_t$, we would expect that the number of scheduled part-timers would always be the maximum allowable $(1 - \gamma)\% = 20\%$ of all scheduled personnel in the optimal solution. Moreover, since $C_o > C_{OT}$, we would expect that overtime would be preferred to temps in the optimal solution, subject, of course, to the bound on the expected amount of overtime allowed.

In Table 1 we give optimal solutions of P_2 : N_1 , N_2 , \bar{N}_3 , \overline{OT} and expected cost as a function of α .

Our intuitions regarding properties of the optimal solution are confirmed from these results. Also as expected, when α increases, N_1 and N_2 increase, while \bar{N}_3 and \overline{OT} decrease. The expected cost is seen to be an increasing function of α , reflecting a growth in scheduled personnel faster than the reduction in temps and overtime, even allowing for the favorable hourly cost differential between scheduled personnel and both overtime and temps.

Recall that “problem P_1 is essentially problem P_2 without the α constraint.” So, for example, the above data when applied to P_1 yield a single optimal solution. Thus, solving P_1 for these data gives $N_1 = 41.6$, $N_2 = 10.4$, $\bar{N}_3 = 2.159$, $\overline{OT} = 28.2$ and an expected cost of 9434. The implied α for this problem is 0.1059, meaning that up to approximately 90% of the days management can utilize temps and/or overtime. When we change only the hourly cost of temps from $C_t = 30$ to $C_t = 25$ we again obtain $N_1 = 41.6$ and $N_2 = 10.4$; but now $\bar{N}_3 = 5.689$ and $\overline{OT} = 0$, with total expected cost reduced from 9434 to 7291. The implied α is again 0.1059. Finally, when we again reduce the hourly cost of overtime, this time to $C_o = 16$ (lower than the cost of either of the two scheduled types of worker), we get, not unexpectedly, $N_1 = 0$, $N_2 = 0$, $\bar{N}_3 = 50$, and $\overline{OT} = 0$, with an expected cost of 6400 and an implied α equal to 0. It is not difficult to perform the calculations for other sets of parameter values, with the management/labor implications of each readily understood.

Table 1. N_1 , N_2 , \bar{N}_3 , \overline{OT} and expected cost for $\alpha = 0.05, \dots, 0.95$

α	N_1	N_2	\bar{N}_3	\overline{OT}	Expected cost*
0.05	40.03	10.01	3.45	32.78	9412
0.1	41.70	10.42	2.16	28.25	9455
0.2	43.59	10.89	1.02	27.07	9481
0.3	44.67	11.16	0.597	22.21	9499
0.4	45.97	11.49	0.261	17.77	9552
0.5	47.05	11.76	0.129	13.78	9626
0.6	48.15	12.04	0.059	10.22	9728
0.7	49.48	12.37	0.056	6.41	9885
0.8	50.82	12.70	0	4.15	10073
0.9	52.63	13.15	0	1.92	10368
0.95	54.46	13.61	0	0.791	10696

*Units are dollars per day of operation.

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